

## CRITICAL SPEED ANALYSIS FOR DUAL ROTOR SYSTEM USING FINITE ELEMENT METHOD

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Intershaft bearing is widely adopted in dual rotor turbofan engines. Since this kind of dual rotor system has two different rotor speeds and the intershaft bearing leads to the coupling between HP rotor and LP rotor, the calculation of the critical speeds is much more complicated than that of rotor systems without coupling. In the paper, a finite element rotor model is established for calculating the critical speeds of a high-bypass turbofan jet engine with intershaft bearing. Assuming that the critical speed ratio of two rotors is a fixed number, in this paper we calculate the critical speeds using three methods based on the finite element rotor model. For the first method, the system critical speeds are obtained directly by the Campbell diagram method. For the second method, the synchronous unbalance response analyses are carried out and the critical speeds were derived from the amplitude-frequency curves. For the last method, multiple group Campbell diagram analyses are conducted. With one rotor rotating speed fixed at constant rpm N, we change the speed of the other rotor to obtain one group of critical speeds. By varying speed N of the two rotors, a critical speed data set can be obtained and plotted as a dual rotor critical speed map. Compared to a single rotor system, the dual rotor system has more critical speeds, which can be classified as critical speeds excited by HP rotor and that by LP rotor. The study shows that the system critical speeds calculated from above three methods are identical. For the first two methods, the speed ratio of two rotors must be a known and fixed number, which is impossible in reality. The third method has no speed relation restriction, and therefore is recommended for analyzing the critical speeds of the dual rotor system.

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Key words: dual rotor, intershaft bearing, critical speed

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### 1. Introduction

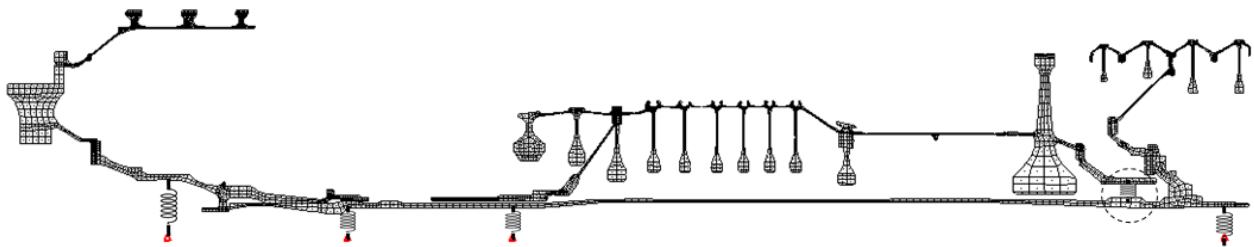
Modern turbofan engine usually has dual or even triple rotors. Some of them have intershaft bearing between inter rotor and outer rotor. This design minimizes shaft deflections caused by the rotor unbalance and improves the engine efficiency, performance and reliability by eliminating the static support structure in the aerodynamics flow path. It also facilitates an easy mounting of the engine casing and gives rise to compactness in the overall structure[1].

Critical speed is defined as the operating speed, at which the excitation frequency of rotating system equals the natural frequency[2]. Critical speed is directly related to engine vibration, and the analysis and design of critical speed is of great importance in aero-engine design.

The transfer matrix methods have been used in the past to deal with multiple rotor-bearing systems. However, due to transfer matrix method's assumptions, it sometimes results in numerical stability problems or root missing problems[3]. The finite element method for rotordynamics was first developed by Nelson and McVaugh in 1976[4]. A study of a twin-spool aircraft engine is also presented in the book of Lalanne and Ferraris[5]. With the advances in computing science, more and more rotordynamics problems are solved using finite element method nowadays.

The aero-engine in this study is a twin spool turbofan engine with intershaft bearing. As shown in Figure 1, the outer rotor operates at high speed and is called High Pressure rotor (HP rotor), while the inner rotor which is also called Low Pressure rotor (LP rotor) works at a relatively low speed. The LP rotor consists of Fan, Low Pressure Compressor (LPC), LP shaft and Low Pressure Turbine (LPT); The HP rotor includes High Pressure Compressor (HPC), drum shaft and High Pressure Turbine (HPT). The engine has 5 bearings and the 4<sup>th</sup> bearing is an intershaft roller bearing.

Since dual rotor system has two different rotor speeds and also the intershaft bearing causes the coupling of the two rotors, the critical speed calculations are much more complicated than the single rotor system. For a dual rotor system, the two rotors will have their own unbalance force excitations. And due to the coupling effect, the critical speeds of a dual rotor system are usually classified as critical speeds excited by HP rotor and that by LP rotor.



**Figure 1.** The finite element model of the rotor system.

## 2. Finite element model of the rotor system

The finite element model of the dual rotor system is established in ANSYS 13.0 software which takes account of gyroscopic moment and rotary inertia. For reducing the size of the model, the shaft and disks are meshed using general axisymmetric element SOLID272. The SOLID272 elements have their base nodes on the master plane. Based upon these master nodes of 2D mesh, nodes required for the 3D mesh are automatically created in the circumferential direction using axis of symmetry of the rotor. The nodes thus generated are called Fourier nodes. The blades are simplified to concentrated masses (Mass21 element), and the blade mass and moment of inertia are considered. The bearing is modelled as linear isotropic bearing using spring element (Combin14 or Combin214). Assume the system support are asymmetrical, which means  $K_{yy}^i = K_{zz}^i$  (The engine shaft is along x-axis)

## 3. Critical speed calculation

The excitation in rotor may come from synchronous excitation or from asynchronous excitation. The excitation due to unbalance is synchronous with rotational velocity and is named synchronous excitation. At critical speed, the vibration of system may increase drastically. These critical speeds can be determined by plotting a Campbell diagram.

Campbell diagram is a graphical representation of the system frequency versus excitation frequency as a function of rotational speed. It is usually drawn to predict the critical speed of rotor system. A sample Campbell diagram is shown in below Figure 2. The rotational speed of the rotor

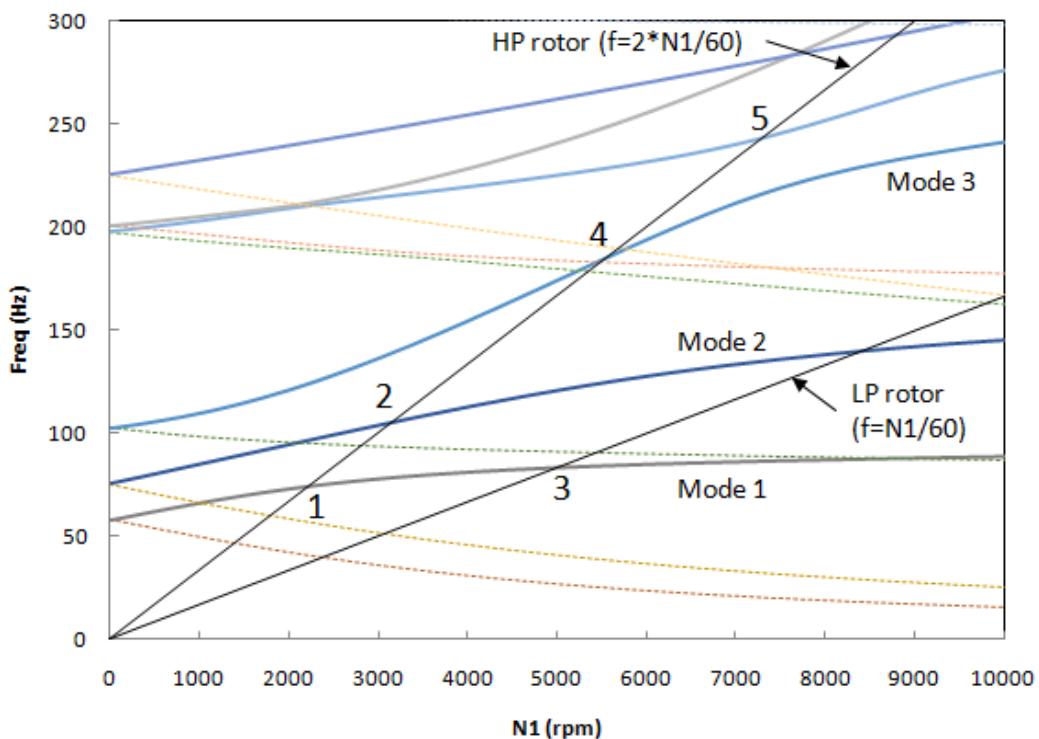
is plotted along the x-axis and the system frequencies are plotted along the y-axis. The system frequencies are extracted for different ranges of operating speed. These frequencies vary along with the rotational speed. The forward whirl frequencies increases with the increase in rotational speed and the backward whirl frequencies decreases with increase in rotational speed. An extra line can be seen in the Campbell diagram, which is called an excitation line, corresponding to the engine rotation frequency usually name engine order 1 and it cross over the modal frequency lines. The critical speeds are calculated at the interference point of modal frequency lines and excitation line.

Critical speed can also be defined as Speeds at which response to unbalance (synchronous whirl) is a maximum[2]. The amplitude of synchronous whirl increases with speed as the critical speed is approached, and then decreases after traversing the critical speed and approaches the value of static imbalance at supercritical speeds. The synchronous response increases as the rotor speed approaches the critical speed and reach a maximum after passing the critical speed[6].

### 3.1 Direct method with Campbell diagram

The direct method with Campbell diagram is usually used for critical speed calculation of a single rotor. The QR damped method is used in the study for extracting system frequencies. The QR damped eigenvalue extraction method is a much more efficient method of obtaining the frequencies for modal analyses involving damping. It combines the best features of the real Eigensolution method and the Complex Heisenberg method. When performing the modal analysis based on QR damped method, the rotor rotating speed should be applied to the rotor. Thus for a dual rotor system, the rotating speed relationship has to be given.

In this study, we assume the rotating speed ratio of HP rotor and LP rotor to be 2. If  $N_1$  is used for representing LP rotor rotating speed, and  $N_2$  for HP rotor rotating speed, the speed relationship can be expressed in  $N_2=2N_1$ . The maximum rotating speed for  $N_2$  speed is set to 18000rpm, and the speed interval is 100rpm.



**Figure 2.** The computed Campbell diagram with  $N_2=2N_1$ .

The computed Campbell diagram is given in Figure 2. The forward modes are presented by thick solid lines, and the thin dashed lines denote backward modes. As the rotor system model is asymmetrical, only the forward modes are supposed to respond to the unbalance excitation.

The first 6 critical speeds for forward modes are extracted from the Campbell diagram, as shown in Table 1. For the same order mode shape, the critical speeds excited by HP rotor are lower than those of LP rotor.

**Table 1.** Critical speeds of the dual rotor engine with direct Campbell Diagram method (N2=2N1).

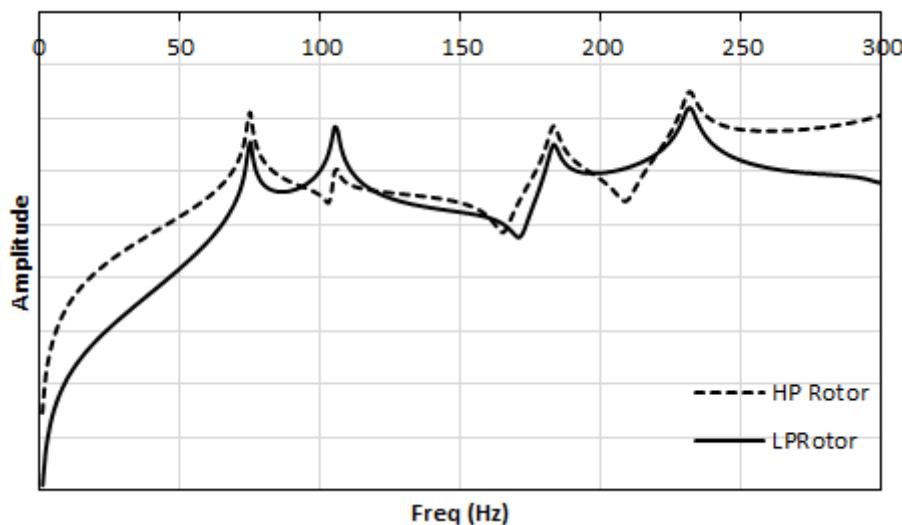
Sequence number	N1(rpm)	N2(rpm)	Excitation rotor
1	2245	4490	HP rotor
2	3166	6331	HP rotor
3	5024	10047	LP rotor
4	5579	11158	HP rotor
5	7329	14657	HP rotor
6	8390	16780	LP rotor

### 3.2 Synchronous response method

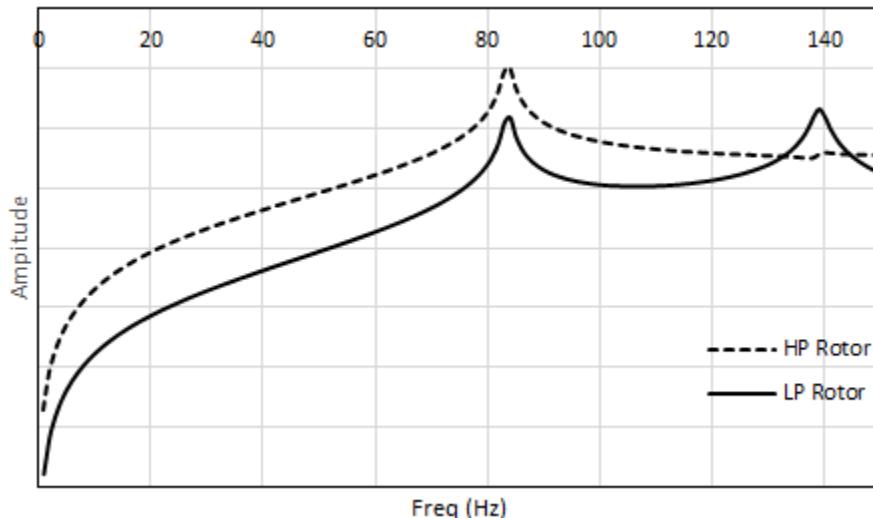
The unbalance of HP rotor is set on the 1<sup>st</sup> stage of HPC, while the unbalance of LP rotor is on the last stage LPT. There are two nodes selected from HPT and Fan, and displacements of the nodes represent the response of HP rotor and LP rotor respectively.

When calculating the unbalance response of the dual rotor system, the speed relationship of the rotors is set to N2=2N1, which is consistent with direct Campbell Diagram method. The maximum N2 is still set to 18000rpm (300Hz), and the frequency interval for analysis is 1Hz.

From Figure 3 and Figure 4, the first 6 critical speeds can be obtained by the peak points. The results are summarized in Table 2.



**Figure 3.** Unbalance response with HP rotor excitation. (Dashed line for response of HP rotor and solid line for response of LP rotor)



**Figure 4.** Unbalance response with LP rotor excitation. (Dashed line for response of HP rotor and solid line for response of LP rotor)

**Table 2.** Critical speeds of the dual rotor engine with synchronous response method ( $N_2=2N_1$ ).

Sequence number	$N_1$ (rpm)	$N_2$ (rpm)	Excitation rotor
1	2250	4500	HP rotor
2	3150	6300	HP rotor
3	5040	8340	LP rotor
4	5520	11040	HP rotor
5	6960	13920	HP rotor
6	8340	16680	LP rotor

### 3.3 Critical speed map method

For the previous two methods of calculating critical speeds of dual rotor system, the speed ratio of rotors is fixed and known. In reality the speed relationship of the two rotors is based on performance calculation, which is not a fixed speed ratio. Critical speed map method does not have any speed ratio limitation.

Figure 5 shows a critical speed map for deriving the critical speed which can satisfy any rotor speed relationship. The dashed line which indicates the aero-engine operating speed curve of two rotors, can be any curve. The intersections of the dashed line with horizontal line are the critical speeds excited by HP rotor, while the intersections of the dashed line with vertical line represent critical speed excited by LP rotor. The procedure of plotting the map is listed below:

- 1) Apply direct Campbell diagram method with fixing  $N_1$  of LP rotor at a constant value while varying  $N_2$  of HP rotor. From the Campbell diagram the corresponding critical speeds can be derived and then listed in Table3.
- 2) In a similar manner, with holding HP rotor rotating speed  $N_2$ , change LP rotor speed  $N_1$ . The corresponding critical speeds are derived and then listed in Table4.
- 3) Plot the data of Table 3 as the horizontal lines, while the data of Table 4 is plotted to vertical lines. The dual rotor critical speed map is obtained as shown in Figure5. According to the map, the system critical speeds are not sensitive to HP rotor rotating speed  $N_2$ .

Lots of calculations which are generated at step 1 and step 2, can be automatically accomplished by ANSYS APDL programming.

The critical speeds for  $N_2=2N_1$  can be derived from the intersections (Marked by Arabic numerals) between engine operating speed curve and horizontal (HP rotor excitation)/vertical (LP rotor

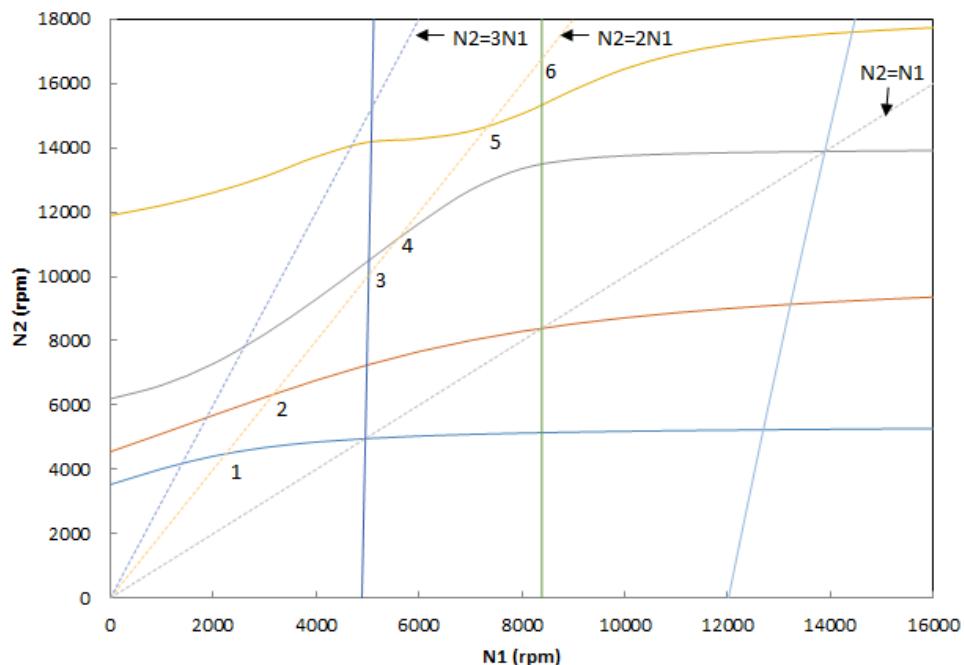
excitation) lines, which is shown in Table 5. Using the critical speed map, it is very easy to compare the critical speeds with different speed relationship between rotors. According the intersections between speed ratio lines and horizon lines, the critical speed decreases with the speed ratio increasing.

**Table 3. Critical speeds (CS) of dual rotor system with N1 fixed.**

N1(rpm)	CS (rpm, 1 <sup>st</sup> mode)	CS (rpm, 2 <sup>nd</sup> mode)	CS (rpm, 3 <sup>rd</sup> mode)	CS (rpm, 4 <sup>th</sup> mode)
0	3529	4550	6197	11896
1000	4018	5113	6617	12211
2000	4413	5682	7287	12607
3000	4682	6241	8203	13106
4000	4853	6771	9291	13723
5000	4963	7250	10468	14174
6000	5037	7664	11648	14280
7000	5090	8010	12687	14520
8000	5130	8294	13353	15054
9000	5161	8526	13636	15798
10000	5186	8716	13755	16454
11000	5206	8873	13815	16915
12000	5223	9004	13852	17216
13000	5238	9114	13876	17416
14000	5250	9209	13893	17556

**Table 4. Critical speeds (CS) of dual rotor system with N2 fixed.**

N2(rpm)	CS (rpm, 1 <sup>st</sup> mode)	CS (rpm, 2 <sup>nd</sup> mode)	CS (rpm, 3 <sup>rd</sup> mode)	CS (rpm, 4 <sup>th</sup> mode)
0	4893	8390	12019	17718
1000	4907	8390	12145	17723
2000	4920	8390	12273	17727
3000	4933	8390	12403	17732
4000	4946	8390	12534	17737
5000	4960	8390	12666	17742
6000	4972	8390	12799	17747
7000	4985	8390	12934	17753
8000	4998	8390	13069	17759
9000	5011	8390	13206	17765
10000	5023	8390	13345	17772
11000	5035	8390	13484	17780
12000	5048	8390	13624	17788
13000	5060	8390	13765	17796
14000	5072	8390	13907	17805
15000	5084	8390	14049	17815
16000	5096	8390	14193	17825
17000	5107	8390	14337	17837
18000	5119	8390	14481	17849



**Figure 5. Dual rotor critical speed map.**

**Table 5.** Critical speeds of the dual rotor engine with critical speed map method (N2=2N1).

Sequence number	N1(rpm)	N2(rpm)	Excitation rotor
1	2251	4502	HP rotor
2	3155	6310	HP rotor
3	5013	10025	LP rotor
4	5599	11198	HP rotor
5	7334	14668	HP rotor
6	8385	16770	LP rotor

## 4. Conclusions

In this study, the critical speed analysis methods for a dual rotor system are introduced and a calculation case with rotor speed ratio of 2:1 is carried out using these three approaches.

1. According to the calculated critical speeds data, the maximum error between different methods is less than 6% (most are less than 1%), which means we get identical critical speeds using these methods for a dual rotor system with fixed rotor rotating speed ratio.
2. The critical speed map method can be used to calculate the critical speeds even without knowing the rotor rotating speed relationship, which is more practical than the other two methods. With the critical speed map, the critical speeds for different speed relationship can be derived directly without any more calculation work.
3. Due to the coupling between the rotors, there may be more critical speeds for the same order mode shape for a dual rotor system, which increases the complexity of rotordynamics design.

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